Three Dimensional Space Frame Models For Bridges

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ABSTRACT: Plane grillage models are widely used for the design of bridges, and are suitable where bridges can be characterised as having a beam and slab arrangement, but have limitations for box girder sections, where a three dimensional analysis is highly preferable. This is normally done with finite elements, which, compared with grillage models, demand considerable design time to convert what are generally stress outcomes to design action parameters, such as moment and shear envelopes. This paper shows how a three dimensional arrangement of members, similar to those which would be used in a plane grillage, can provide direct design actions for virtually any bridge type. The method has similar accuracy to a good finite element model, and requires considerably less computer power. The paper also proposes a simple approach for treating shear lag. Both the shear lag analysis, and the three dimensional grillage method are very "transparent" about the interactions of the various parts of the structure carrying the loads, and both could prove useful as a teaching aid. Finally, experience using the method to analyse the 9 span 660m long Mt Henry Bridge is discussed.

1 INTRODUCTION

1.1 *Box Girders*

The box girder is an efficient form of construction, especially for larger bridges, because it minimises weight while maximising flexural stiffness and strength. Single cell, multi-cell, and multiple box girders are all commonly used forms. However, the behaviour of box girders under eccentric loading is complex, because although the box's shear torsional stiffness is readily calculable, much larger flexural twisting distortion is caused by in plane flexure of the four sides of the box. Three dimensional analysis is required to accurately model this distortional behaviour.

1.2 *Finite Element versus WW Space Frame Analysis*

For three dimensional analysis, finite elements are normally used, but the models are time consuming to set up, difficult to modify, and generally give stress outputs which need to be converted to moments and shears for design purposes. This paper will describe a three dimensional space frame, which the authors' believe is a new method, and which will be referred to as a WW Frame (after the authors' initials). A more detailed discussion is given in Wenham & Wyche (2004). The WW Frame is based on the principles of plane grillage analysis, and can be used to accurately model the complex behaviour of box girders, while retaining the simplicity and usefulness associated with plane grillages. To further refine the analysis, a new method for quantifying the effects of shear lag is also presented.

1.3 *Other Analysis Methods*

Several other space frame modelling techniques are discussed in Hambly (1990). These include the Truss Space Frame, McHenry Lattice, and Cruciform Space Frame, which is most similar to the WW Frame. Cruciform models can be quite accurate, even when relatively coarse divisions are used, however, as described in Hambly (1990), some interpretation of the results to obtain design actions is required. Other methods, also described in Hambly (1990) include:

- Beams-on-elastic-foundations methods can be attempted but tend to be complex for anything other than simple structures.
- Plane (two dimensional) grillage analysis is commonly used within the industry for beam and slab type bridges. Hambly (1990) suggests methods for approximating large cell box sections, but the methodology is clumsy, time consuming and error prone.
- For multiple discrete cell box girders such as Super T's, it is possible to use plane grillages to

model each box girder as a single beam with a bending, shear and torsion stiffness. However approximations must be made to reasonably model the actual torsion (or twisting) stiffness of each beam. Each beam must also be small enough relative to the size of the structure, so that its webs are close enough together to be considered as acting equally in flexure within that individual beam.

- For large cell single or double (or more) box girder bridges, a torsion and flexure line beam or grillage approach is sometimes used, but such models inevitably give wrong results. This is because even if the shear torsion stiffness is artificially reduced so the model gives correct deflections, each beam will still output equal web moments, and all eccentric loading will have to be carried entirely by torsion shear. This is quite a different mechanism from the substantial contribution of in plane web flexure which actually occurs. Note that the extra twisting distortion caused by in plane flexure of the walls can be an order of magnitude more than by the shear torsional stiffness alone (although bracing reduces this discrepancy).

All these approaches are approximate at best and do not transparently show the designer how the structure is actually behaving.

2 THE WW SPACE FRAME THREE DIMENSIONAL ANALYSIS

2.1 *The Concept*

The WW Frame analysis method was developed to accurately model the complex behaviour of box girders, while still achieving the simplicity and usefulness associated with two dimensional grillage methods. Refer to Figure 1. The basic idea is to convert a plane grillage to three dimensions by splitting each box into:

- Two beams placed on the web lines at the gross section centroid depth (Beams A and B in Figure 1). There would be three such beams for a two cell box girder etc
- One beam on the line of the deck slab at the gross section horizontal centroid (Beam C in Figure 1). There will only ever be one such beam because every bridge has only one deck.
- One beam placed on the line of the lower flange slab at the box centroid (Beam D in Figure 1). For bridges with more than one box girder, there would be one such beam placed at the centroid of each discrete girder.

These beams then have the following stiffnesses:

- A and B have a vertical stiffness, but near zero lateral stiffness (see also paragraph 2.2).
- C and D have a lateral stiffness but near zero vertical stiffness (see also paragraph 2.2).
- The total vertical stiffness of A+B then equals the gross section vertical stiffness (corrected as discussed later for shear lag).
- Similarly the total horizontal stiffness of C+D equals the gross section horizontal stiffness.
- The axial stiffness (area) of A,B,C and D is zero, because it is already modeled in the flexural stiffness. It would be necessary to incorporate axial stiffnesses to model other structural types such as haunched or arch bridges, or multi-storey buildings, but such models have not yet been attempted by the authors.

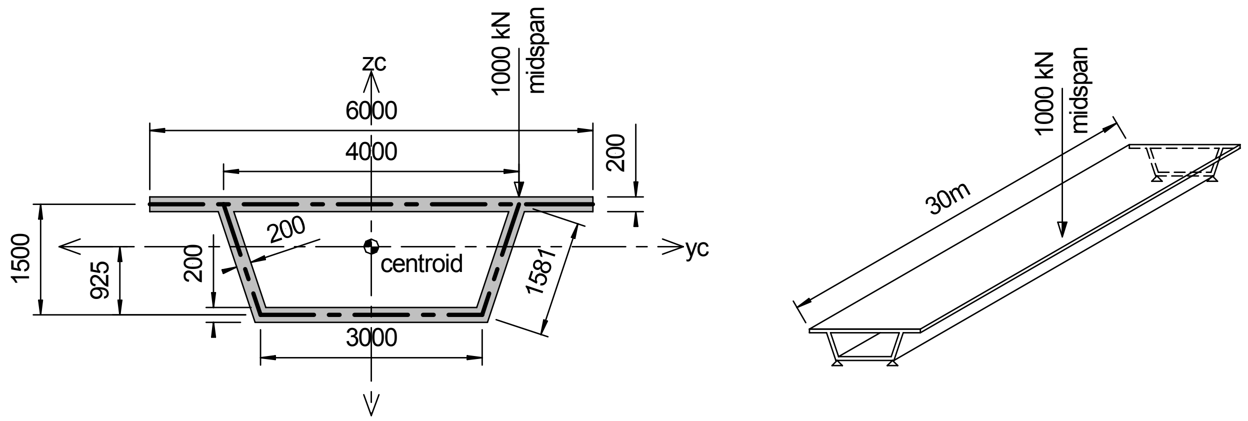
The transverse connection of these beams and modeling of the transverse behaviour of the webs and slabs is as follows:

- The curvatures of each of these beams A, B, C, and D project strains out from their centriodal location via a "fishbone" arrangement of perpendicular members.
- These perpendicular members are very stiff in the plane of bending between their parent beam and the point to which they project the strain.
- That point is the corner where they are joined to a similar member from another of the longitudinal beams to form the transverse frame of the box.
- For bending out of plane (about their weak axis), they have their true flexural stiffness, and also their true area, and therefore correctly model the transverse stiffness of the webs and flanges of the box.

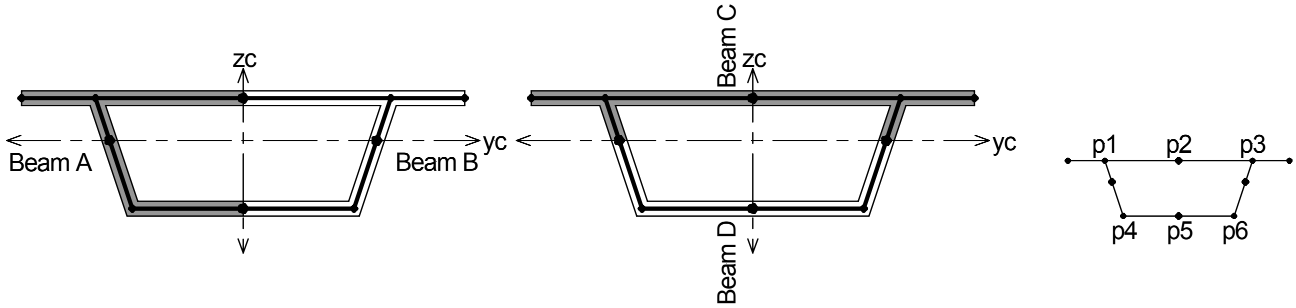
2.2 *Rules to Set Up a WW Frame Model with Worked Example*

A sequence of rules will be given as the worked example in Figure 1 is simultaneously presented. For that section, the gross properties are:

- Area (A) = 2.390m^2
- Moment of inertia about the horizontal (y) axis (I_y) = 1.045m^4
- Moment of inertia about the vertical (z) axis (I_z) = 5.841m^4
- Shear torsional stiffness about the longitudinal (x) axis (J_x) = 2.170 m^4 (not used in WW Frame)
- Modulus of elasticity (E) = 32000 MPa
- Poisson's ratio (ν) = 0.2
- Shear modulus (G) = 13333 MPa



DETAILS OF WORKED EXAMPLE

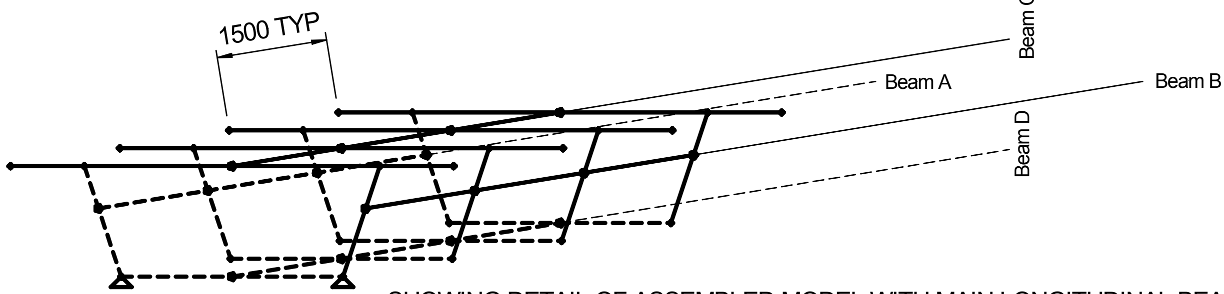


Longitudinal Beams A & B representing vertical stiffness

Longitudinal Beams C & D representing horizontal stiffness

Frame Deformation Identification Points

SHOWING MAIN LONGITUDINAL BEAMS AND JOINING TRANSVERSE FRAME



SHOWING DETAIL OF ASSEMBLED MODEL WITH MAIN LONGITUDINAL BEAMS AND JOINING TRANSVERSE FRAMES

From these values and the dimensions in Figure 1, the following WW Frame properties were derived:

- Beams A and B:

  (i) Area (A) = 0 (Rule: must be zero because flexural stiffnesses of C and D already include this area).

  (ii) Shear area (A_s) = area of web = 1.581 x 0.2 = 0.316m^2.

  (iii) Bending stiffness about horizontal (y) axis (I_y) = ½ x (Iy gross, 1.045m^4) = 0.552m^4 (could be modified for shear lag – discussed later).

  (iv) Bending stiffness about vertical (z) axis (I_z) = (1.5 x 0.2^3)/12 = 0.001m^4 (Rule: Must include, although theoretically zero (see paragraph 2.1), because this stiffness distributes vertical loads along sloping web. So small compared with gross lateral stiffness that it can correct this local effect without affecting global model.)

  (v) Torsion stiffness about longitudinal (x) axis (I_x) = assumed 0.

- Beam C:

  (i) Area (A) = 0 (Rule: must be zero because flexural stiffnesses of A and B already include this area).

  (ii) Shear area (A_s) = area of top flange = 6.000 x 0.2 = 1.200m^2.

  (iii) Bending stiffness about horizontal (y) axis (I_y) = (6.0 x 0.2^3)/12 = 0.004m^4 (Rule: Must include, although theoretically zero (see paragraph 2.1). This longitudinal slab member helps to distribute local wheel loads round the deck slab, just like a plane grillage. More (zero area) members of this type may be added, even in cantilevers, to refine the local deck slab analysis, but their combined stiffness will still be so small compared with gross vertical stiffness, that they will not affect the global model.)

(iv) Bending stiffness about vertical (z) axis (I_z) = 4.314m^4 (Choice of centroid to split C and D somewhat arbitrary. See discussion on shear lag later).

(v) Torsion stiffness about longitudinal (x) axis (I_x) = torsion stiffness of continuous slab = (4.0 x 0.2^3)/6 = 0.005m^4 (see Hambly (1990)). Done as part of deck grillage carrying wheel loads.

- Beam D:

(i) Area (A) = 0 (Rule: must be zero because flexural stiffnesses of A and B already include this area).

(ii) Shear area (A_s) = area of bottom flange = 3.000 x 0.2 = 0.600m^2

(iii) Bending stiffness about horizontal (y) axis (I_y) = (3.0 x 0.2^3)/12 = 0.002m^2 (Rule: Must include although theoretically zero (see paragraph 2.1). Similar to A, B, and C, although usually less issues with local effects).

(iv) Bending stiffness about vertical (z) axis (I_z) = 1.527m^4 (See comment for Beam C above. Note I_z for Beam C (4.314) + I_z for Beam D (1.527) = I_z gross = 5.842m^4)

(v) Torsion stiffness about longitudinal (x) axis (I_x) = assumed 0.

- Intermediate frame members, at 1.500m intervals with webs and slabs all having same thickness and therefore same properties:

(i) Area (A) = area of slab = 1.5 x 0.2 = 0.3m^2.

(ii) Shear area (A_s) = rigid (large) (Rule: must be very large to project the strains from Beams A, B, C, and D with high accuracy).

(iii) Out-of-plane bending stiffness (I_y) = (1.5 x 0.2^3)/12 = 0.001m^4.

(iv) In-plane bending stiffness (I_z) = rigid (large) (Rule: must be very large to project the strains from Beams A, B, C, and D with high accuracy).

(v) Torsion stiffness (I_x) = assumed 0, except in deck slab carrying wheel loads use torsion stiffness of continuous slab = (1.5 x 0.2^3)/6 = 0.002m^4 (see Hambly (1990)).

3 RESULTS OF ANALYSIS

3.1 *Analyses Performed*

With reference to the Frame Deformation Identification Points in Figure 1, Table 1 provides results for comparing the following analyses:

- T&F: A torsion and flexure line beam analysis using gross section properties, I_y = 1.045m^4 and J_x = 2.170 m^4.
- FE: Finite Element, required 1440 rectangular plate elements, and 1464 nodes.
- WW: WW Frame, required 310 members and 210 nodes.
- FE+D: Finite Element with end diaphragms restraining all sides of the box.
- WW+D: WW Frame with end diaphragms restraining all sides of the box.

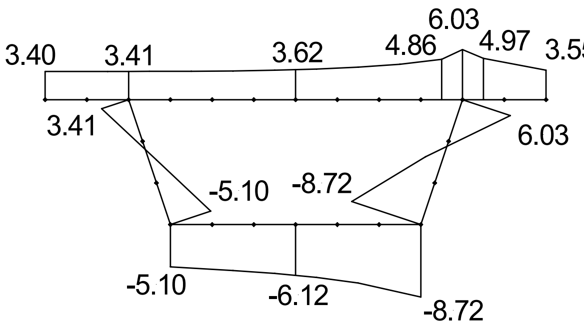
Table 1. Results of Analyses

Analysis	T&F	FE	WW	FE+D	WW+D
vertical deflection					
midspan					
p1 mm	-16.5	-12.7	-12.8	-16.3	-15.8
p2 mm	-17.5	-17.9	-17.4	-17.8	-17.3
p3 mm	-18.5	-23.4	-22.6	-19.7	-19.4
p4 mm	-16.7	-13.6	-13.7	-16.4	-16.0
p5 mm	-17.5	-17.9	-17.7	-17.8	-17.6
p6 mm	-18.3	-22.3	-21.6	-19.4	-19.1
end					
p1 mm	0	+1.6	+1.3	0	0
p2 mm	0	-0.1	-0.4	0	0
p3 mm	0	-1.9	-1.4	0	0
p4 mm	0	0	0	0	0
p5 mm	0	-0.0	+0.2	0	0
p6 mm	0	0	0	0	0
horizontal deflection					
midspan					
p1 mm	+0.3	+4.5	+3.5	not recorded	
p2 mm	+0.3	+4.6	+3.5		
p3 mm	+0.3	+4.7	+3.6		
p4 mm	-0.5	+1.8	+1.0		
p5 mm	-0.5	+1.8	+1.0		
p6 mm	-0.5	+1.7	+1.0		
end					
p1 mm	0	+4.7	+3.8	0	0
p2 mm	0	+4.7	+3.8	0	0
p3 mm	0	+4.7	+3.8	0	0
p4 mm	0	0	0	0	0
p5 mm	0	+0.0	+0.0	0	0
p6 mm	0	0	0	0	0
midspan					

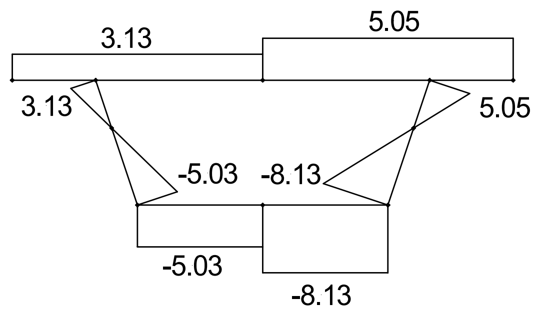
torsional rotation rad 10^{-6}	520	2670	2430	840	900
rotation rel to T&F model	1.0	5.1	4.7	1.6	1.7

3.2 *Flexural Stress Comparison*

A comparison of the flexural stresses from the Finite Element analysis and the WW Frame is given in Figure 2. The WW Frame stresses could be adjusted slightly further if the lateral bending stresses are added, and become even closer to the shape of the Finite Element results.



Midspan Axial Stress (MPa)
Finite Element Model



Midspan Axial Stress (MPa)
WW Frame Model

4 SHEAR LAG AND ALLOCATION OF STIFFNESS BETWEEN BEAMS

4.1 *Calculating Shear Lag with Finite Elements*

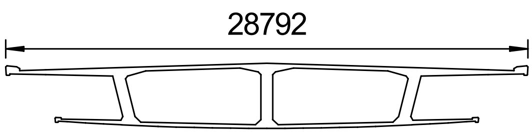
Allowance for shear lag is often made by using reduced "effective flange widths", which result in lower flexural stiffnesses, re-producing the increased curvatures and peak stresses caused by shear lag. The choice of "effective width" really depends on which particular non-eccentric load pattern is chosen. Concentrated (point) loads (or reactions) tend to produce more pronounced shear lag than loads causing more uniform curvature along the beam. For most concrete sections Finite Element analysis shows that the effects of shear lag in a global analysis sense are minor, and a simple way to test this is to subject the beam Finite Element model to a simultaneous forced uniform deflection of both webs (or all, if more than two) at the midspan. This is a severe load pattern for shear lag. Instead of then painstakingly extracting stresses and determining the "effective width", it is much simpler to compare the total sum of the web deflection point forces with the reaction of a line beam using gross section properties, for the same forced deflection. The percentage difference represents the global reduction in stiffness to account for shear lag. If this was a significant difference, a more sophisticated investigation may be justified, but for cases so far analysed by the authors, reduced stiffness is at least 95% of gross stiffness.

4.2 *Allocation of Shear Lagged Stiffness*

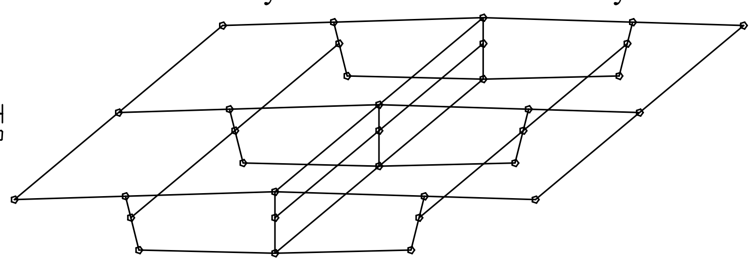
This procedure may be applied in both the vertical and horizontal directions, and can also be used allocate stiffnesses between unsymmetrical beams, while simultaneously allowing for shear lag (e.g. Beams C and D in Figure 1). For example, if a single cell box has a gross transverse stiffness of $10m^4$, for 100mm lateral deflection it may require 65000 kN. The Finite Element model may give loads of 38400 kN and 24000 kN for the top flange and lower flange respectively, for simultaneous lateral deflections of 100mm. This means that the gross effective lateral stiffness is (38400 + 24000 = 62400) / 65000 x 10.0 = $9.6m^4$, and it will be allocated (38400 / 65000) x 10.0 = $5.9m^4$ to the top flange, and $3.7m^4$ to the lower flange.

5 MOUNT HENRY BRIDGE, PERTH

The strengthening design for Mount Henry Bridge in Perth is an example where a complex structure has been analysed with a WW Frame. Wyche Consulting



MOUNT HENRY BRIDGE
TYPICAL SECTION



PART WW FRAME MODEL

initially developed the method for this project, which required strengthening and widening of the existing two cell box girder bridge for two new railway lines. To model the nine span 660m long bridge, a five span WW Frame model of the existing superstructure was set up (Figure 3), which was checked and calibrated against a single span Finite Element model. The WW Frame model provided a tool for quick analysis of the structure, which allowed the client to investigate a number of construction options during the tendering period.

6 CONCLUSION

A new three dimensional space frame analysis method, the WW Frame, has been developed which has the following features:

- It can be used to quickly and accurately model the complex three dimensional twisting distortional behaviour of box girders. The only other commonly used, feasible, reliable method is Finite Elements.
- It is similar in principle to plane grillage analysis, and gives direct design actions such as moment and shear force envelopes, which is much more practical than Finite Element models.
- WW Frame models require many fewer members than Finite Element models, meaning that large complex structures can be more accurately modelled within the limitations of even small analysis programs.
- A simple method based on a targeted use of Finite Elements is described for assessing shear lag. This not only calibrates the WW Frame models for shear lag, but also provides a simple rational way of allocated gross section stiffnesses to the various beams which make up the WW Frame.
- Both the WW Frame and the shear lag analysis provide a clear insight for the designer (or student) to the structural behaviour of the box girder.

ACKNOWLEDGMENT

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Figure 1. Setup of WW Frame with Worked Example.

Figure 2. Comparison of Finite Element and WW Frame Flexural stresses.

Figure 3. Mt Henry Bridge WW Frame Model.